

Quantum theory of microworld and the reality

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Abstract

The discoveries in the end of the 19th century and in the beginning of the 20th century have paved the way for the physical science of the new era. It was shown that the hitherto basic stones (atoms) of the matter world were not quite stable and did not represent the smallest matter objects. And especially, that the energy of the light was not transmitted continuously but in small quanta.

The mathematical model of orthodox quantum mechanics was accepted by the physical community as the only theory of microworld, even if it involved a series of strange (paradoxical) characteristics. It is possible to say that the things started to change in the end of the 20th century. More attention has been devoted to the problem of time reversibility and to the question whether it is not possible for the microworld to exhibit irreversible evolution similar to that of macroscopic world.

To understand better these physical problems and the contemporary theoretical attempts it is necessary to follow at least shortly the most important points of the whole physical story since the end of the 19th century. It will be shown that one of the basic quantum-mechanical assumptions (physical interpretation of the general mathematical superposition principle) has been introduced without any actual reason and experimental tests; some misleading arguments having been used, in addition to.

The difference between microscopic and macroscopic worlds diminishes significantly if the quantum-physical model starts fully from time-dependent Schrödinger equation and the general superposition principle linking states with different physical properties is abandoned; divers physical meanings of different initial conditions of individual solutions being fully respected. Such a model (including suitably extended Hilbert space) may be regarded as a generalization (or even as a mere quantum adaptation) of the classical approach. The physical systems with both fixed and changing numbers of objects may be now described in the framework of one common mathematical model.

1 Introduction

It is possible to say that the time flow (time evolution) has represented important problem in quantum mechanics practically during the whole past century. We shall start, therefore, in this introduction with a brief survey of the beginnings of quantum mechanics. This beginning is linked closely with the name of E. Schrödinger [1], who believed firmly in wave nature of matter. He proposed his famous wave equation in 1925 and showed that it was possible to reproduce all main results obtained earlier on the basis of Hamilton equations. All characteristics of a physical system could be then derived with the help of wave function $\Psi(\mathbf{r}, t)$ where \mathbf{r} represented coordinates of all matter objects. A great success has been seen in that in the case of bound systems the discrete atom energy levels have corresponded to eigenvalues of corresponding Hamiltonian [2]. The wave function Ψ was interpreted as probability distribution by M. Born [3]. N. Bohr (see [4]) attributed then the probabilistic properties directly to individual matter particles; Heisenberg's [5] uncertainty relations being linked with them.

A further step was done by J. von Neumann [6] who showed that individual Ψ -functions at given t values may be represented by vectors in the Hilbert space spanned on eigenfunctions of the corresponding Hamiltonian (determining the total energy of a given physical system). The additional assumption has been then introduced into the corresponding model: general mathematical vector superpositions have been interpreted as physical relations between divers physical states. And Pauli's criticism formulated already in 1933 has concerned just it. Pauli [7] has showed that under the mentioned conditions a correct representation of time evolution (introduction of the time operator T in the Hilbert space) has required for the Hamiltonian to possess continuous spectrum belonging to the whole real interval $(-\infty, +\infty)$, which contradicts the necessity of energy being positive (or at least limited from below). It contradicts in principle also the characteristics following from the corresponding time-dependent Schrödinger equation.

Pauli's problem has been, however, related later to another one concerning the non-unitarity of exponential phase operator

$$\mathcal{E} = e^{-i\Phi}$$

where the phase $\Phi = \omega T$ is proportional to time. The exponential phase operator for linear harmonic oscillator was defined by Dirac [8] already in 1927 and it was assumed to be unitary. But it was shown later by Suskind and Glogower [9] that the operator \mathcal{E} defined in such a way was isometric only (as it will be shown in Sec. 3.13); see also the review by Lynch [10]. This fact and Pauli's critique have been regarded all the time as following from one common source. However, both these problems will be shown to be solved only if they are handled as two different problems.

In Part 4 we will present not only the solution of both the problems but also that the new quantum-theoretical model may be brought to harmony with available

experiments (including those having been performed in the last ten years). However, to provide necessary insight into the related problems the contemporary status of corresponding physical knowledge will be summarized, first. In Part 2 we shall discuss the state of physical research in the end of 19th century devoting main attention to problems preparing the era of modern physics. The problems concerning the contemporary quantum physics will be then explained in Part 3; with the respect to points being solved in Part 4. Final summary will be given in Part 5.

2 Physics in the end of the 19th century

2.1 Light waves and matter objects

One can say that till the end of the 19th century two different worlds existed one beside another: matter objects and the light or electromagnetic waves. The behavior of the formers was ruled by the laws that were formulated originally by G. Galilei and I. Newton. In the 19th century they were then described systematically, e.g., with the help of Hamilton equations; W. Hamilton (1805-65).

The other part (light and electromagnetic waves) of the observable world was governed by equations of J. Maxwell (1831-79) who unified the description of electric and magnetic phenomena in 1865 [11]. The relations of electromagnetic waves to matter world were given by the electric and magnetic properties of matter objects. These relations were studied on the basis of different frequency and temperature dependencies, especially with the help of the so called black body radiation.

In both the cases the nature phenomena were described with the help of differential equations that provided fully causal and practically deterministic behavior of all nature processes. The evolution of any physical system was (in principle) uniquely determined when its state was known fully at one time instant.

2.2 Atoms and molecules

The assumption that matter objects consisted from some not further divisible particles was formulated already by Democritos (460-370 BC) in the old Greece. In the modern age it was formulated again by P. Gassendi (1592-1655) and R. Boyle (1627-1691); and later developed by M. V. Lomonosov (1711-65) and J. Dalton (1766-1844); e.g., the atom weight was introduced and the first table of atom weights proposed by Dalton in 1803. D. I. Mendeleev (1834-1907) compounded then the periodical law of elements in 1869.

Besides the atoms the existence of molecules consisting of two or more atoms were assumed, too. All the conclusions were derived from the results observed with the help of chemical reactions and corresponding laws. Similar conclusions could be drawn from a great amount of various data. And further predicted properties were found to agree with observations. Thus, the existence of atoms was accepted by the

whole physical community even if the atoms and molecules could not be observed directly at that time.

2.3 Thermodynamics of gas systems

The grounds of heat theory on the basis of kinetic energy of atoms and molecules were formulated cca in 1790. The average kinetic energy of atoms (or molecules) was assumed to rise proportionally to the measured temperature.

The molecular theory was taken also as the basis of thermodynamics of gas systems. The basic behavior of such a system consisting of many molecules has been described by the state equation

$$\frac{pV}{T} = \text{const}$$

where V is volume, p - pressure, and T - temperature. The given equation holds when the gas system is in equilibrium. The system has its inner energy U that rises with temperature T :

$$dU = \beta dT.$$

Entropy S of the system has been then defined by

$$dS = \frac{dQ}{T}$$

where Q is heat content; its change being given by

$$dQ = dU + pdV.$$

In an enclosed system the entropy change dS is always positive, the entropy rising to a maximum value when the system reaches its equilibrium state. L. Boltzmann (1844-1906) linked this (deterministic) entropy rise with the tendency of the system to go to uniform average probability distribution of individual molecules [12]. In 1867 he proposed the equation

$$S = k \log w$$

where w is the probability of molecule distribution determined in the classical way.

The distribution probability rise was denoted as a basic natural law, even if it is in principle the result of collision and diffusion processes (and corresponding rules). It is evident that Boltzmann was influenced strongly by the positivistic philosophy (refusing any ontology and metaphysics) as practically all physicists at that time. Agreement of model predictions with measured numerical values was for them quite sufficient to regard corresponding model (hypothesis) as verified; without testing its logical structure.

2.4 Black body radiation

As already mentioned the matter world consisted at that time of two different media: the matter objects moving according to equations of motion and the light (electromagnetic) waves being transmitted by swinging of the ether present in all space. The interactions between the electromagnetic and light waves were described by Maxwell's equation [11]; waves being excited by moving electrically charged particles.

The mutual interactions between heated objects and light were studied with the help of the so-called black body radiation. The frequency spectra at different temperatures inside a body (being quite black at the absolute zero) were measured. In 1896 Wien (1864-1928) showed that all experimental frequency spectra (for higher values of ν) may be described with the help of the formula

$$\rho(\nu) = a\nu^3 e^{-\frac{b\nu}{T}}$$

where ν is the corresponding light frequency, and a, b are parameters adapted according to the material of the black body.

The theoretical formula derived by Rayleigh (1842-1919) and Jeans (1877-1946) on the basis of continuous transfer of energy between the two different objects (body and ether) was equal to

$$\rho(\nu) = \frac{8\pi}{c^3} \nu^2 k_B T,$$

i.e. very different from the experimentally measured spectra.

This contradiction was solved by M. Planck (1858-1947) in 1900 by assuming that the light energy was transferred always in finite amounts and that only multiples of a basic energy amount might be emitted [13]. We will describe his approach in Sec. 2.6.

Planck's quantum idea opened the main way to the physics of the 20th century. The other important way concerned the problem of ether, which will not be discussed in this paper; the problem being mentioned only shortly in the next section.

2.5 Light ether

When the light was transmitted not only through the air but also through different transparent bodies the question arose how the ether was related to matter objects. Is it quite immobile in the whole world space? or: Does it move together with moving matter objects?

Several experimental studies were performed that led to different conclusions. The experiment of Michelson and Morley (1887) led to the conclusion that the ether should be closely linked to the motion of the earth. A quite opposite result could be derived from the measurement of star aberrations; the phenomenon discovered by Bradley in 1727. The ether should be linked to the sun as the observed fixed stars exhibited small ellipses. And according to Fizeau's experiment (1851) the ether

should be partially took along by moving objects when the light velocity changed with the velocity of water.

These three experiments put so different requirements on the ether that its existence should have been doubted. That led Einstein [14] to refuse the existence of ether and to formulate the theory of special relativity in 1905. He assumed that the light velocity depended neither on the velocity of a moving system nor on its actual direction.

We will not discuss this problem further here. We have mentioned the theory of relativity as it belonged to two basic theories of the past century applied to the description of the microscopic world, even if these two theories have provided quite different pictures: special relativity being deterministic as classical physics and quantum mechanics refusing any causality.

2.6 New discoveries at the break of centuries

The way to the new physics was opened mainly by discoveries in the last five years of the 19th century. They were: The X radiation emitted from cathode tubes and discovered by W. Roentgen (1895), ionizing radiation emitted by some heavy substances and found by A. S. Becquerel (1896), and the discovery of electron by J. J. Thomson (1897). These discoveries showed convincingly that there is a very rich physics under the level of our direct observations, which laid quite new claims to measurement techniques. These new discoveries influenced strongly not only the physical research, but they evoked also many new application regions. E.g., quite new branches of medicine arose along with the new branches of physics.

However, as already mentioned the physical thinking was fundamentally influenced by M. Planck [13] who introduced the quantum idea in solving the problem of black body radiation. He assumed that the exchange of energy between an oscillator in the body and an electromagnetic (light) wave might occur only in multiples of a basic energy quantum, i.e., the exchanged energy might equal

$$E_n = nh\nu$$

where ν was radiation frequency and h was the famous constant of Planck; $n = 1, 2, \dots$

He assumed further that the energy of oscillators increased with temperature and one could write for the numbers of states with higher energy

$$N_n = N_0 e^{-\frac{nh\nu}{k_B T}}$$

where $k_B T$ was Boltzmann factor. The average energy per one oscillator in the equilibrium was then

$$\bar{E} = \frac{E_t}{N_t} = \frac{\sum_m N_m m h \nu}{\sum_m N_m}$$

where E_t is total energy and N_t - the number of all eigenoscillations. It holds then

$$\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}.$$

And if one puts this average energy into the formula of Rayleigh and Jeans (see Sec. 2.4) instead of $k_B T$ it is possible to obtain the following final formula [13]

$$\rho(\nu) = \frac{8\pi}{c^3} \frac{h\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}.$$

The last formula passes for very large and very low frequencies into one of the two formulas introduced in Sec. 2.4.

It is possible to say that the discoveries and ideas mentioned in this section formed the new basis from which the physics of microscopic world developed. The problem of photoeffect (discussed in the next section) has belonged to the first phenomena from this series.

3 Physics of the 20th century

3.1 Photoeffect and the birth of duality

Planck assumed that the electromagnetic radiation should have been interpreted as waves and that the cause of discrete energy emission was hidden in oscillating matter objects. There were, however, other experiments that were not in full harmony with such an assumption.

It was found that metals irradiated by the light of higher frequency emitted some charged particles. In 1902-3 Lenard [15] and Ladenburg [16] showed that these particles were electrons. However, the electrons were emitted only if radiation frequency was greater than a minimal value ν_0 . It was found, too, that the energy of emitted electrons did not depend on light intensity, but only on light quality, which could not be interpreted on the basis of classical wave ideas. It followed from this fact that the light energy in the wave front could not be distributed in a continuous and uniform way.

Einstein [17] tried to solve the problem in the following way: He assumed that the energy of electromagnetic wave came in discrete units of $h\nu$ and each of these units occupied a very small volume only. Therefore, he assumed that a radiation (light) quantum had the same properties as a matter particle. Einstein wrote for the photoelectric process the following equation

$$\frac{1}{2}mv^2 = eV = E = h\nu - \Phi$$

where e , m , and v are the electric charge, mass and velocity of the knocked-out electrons; V is the minimal potential for which no emission occurs (boundary of the

photoeffect); and $\Phi = h\nu_0$ is the binding energy of an electron which characterizes the given material (metal). Einstein assumed also in 1904 that the ratio

$$\frac{eV + \Phi}{\nu} = h$$

is independent of the material and of the intensity and frequency of light [17], being equal to the constant that was introduced earlier by Planck.

Thus the duality appeared in physical story for the first time. Particles (photons) were the carriers of energy that belonged to the corresponding electromagnetic field, while the electromagnetic waves around these particles represented other characteristics. The particle structure was necessary to understand the photoeffect, while the waves enabled to explain the interference and diffraction of electromagnetic radiation. The energy of individual photons has equaled $h\nu$.

However, practically any physicist did not believe in the quantum idea at that time. The situation changed in 1914-16 when R. Millikan (1868-1953) confirmed all predictions of Einstein experimentally. Einstein obtained then the Nobel price for theoretical predictions of photon in 1922.

3.2 Atom nucleus

The discovery of electron and the existence of photoeffect changed also our view on the indivisibility of atoms. J. J. Thomson (1856-1940) proposed the new atom structure in 1903. He assumed that the atom was a very tiny sphere the electrons floated in.

A quite new result was then obtained by E. Rutherford (1871-1937) in 1911 [18]. He irradiated a tiny gold foil by a beam of α particles. The scattering characteristics showed that the atom mass was not distributed homogeneously over the whole atom volume but concentrated in a much smaller volume.

It was the basis for atoms to be regarded as small planetary systems: A positively charged heavy nucleus in the center and negative electrons running around. The nucleus was thousand times smaller than the whole atom. There were, however, problems with such a model as according to electromagnetic theory (classical physics) the corresponding systems could not be stable. The electrons running round a nucleus had to emit electromagnetic waves and could not move in stable orbits.

3.3 Atom model

The problem of a stable atom was solved by N. Bohr (1885-1962) with the help of two additional postulates in 1913. He combined the planetary model with the quantum picture of Planck and Einstein. The two postulates were [19]:

1) The postulate of stationary states. According to classical physics the electrons might circulate in any distance from the nucleus. However, Bohr assumed that

only certain tracks with discrete energies were possible and that the values of these energies were

$$E_n = nh = \oint p dq, \quad n = 1, 2, 3, \dots$$

where p is the momentum of electron, q - its distance from the nucleus, and n - the so-called main quantum number.

2) The frequency postulate. If the electron passed from one stationary track with energy E_n to the other one with energy $E_m < E_n$, the atom emitted spontaneously the energy

$$h\nu_{nm} = E_n - E_m.$$

Bohr obtained for these energies:

$$E_n \simeq -\frac{2\pi^2 m_e e^4}{h^2 n^2}$$

where m_e is electron mass, and e - its charge. The frequencies of emitted light by hydrogen atoms corresponded to frequency values that were measured by Balmer (1825-98) already in 1880.

N. Bohr assumed that electrons ran along circle tracks around the nucleus. A. Sommerfeld (1868-1951) generalized these tracks and assumed that also elliptic tracks were possible. He considered relativistic effects, too. Everything was in good harmony with the then experimental data (see e.g. [20]).

It is necessary to introduce that the models of Bohr and Sommerfeld represented a partial success only. The theory described well light frequencies but it was not able to predict corresponding intensities. It was not possible, either, to apply Bohr's postulates to atoms with a greater number of electrons.

Some questions remained unanswered: a cause why in principle classical tracks were limited to some special orbits only. And further, how the electrons pass from one orbit to the other one. These transitions were spontaneous and indeterministic.

Various attempts were done to explain these characteristics in the framework of classical physics. E.g., in 1924 Bohr, Krammers and Slater assumed that the classical physics held also in atom region, but the energy and momentum were not conserved [21]. The idea was refused experimentally by Bothe and Geiger [22] and the conservation laws were confirmed in 1925 by A. Compton [23].

3.4 Asymmetric world

In spite of the just mentioned problems the interpretation of matter objects seemed to be clear: atoms consisted of positively charged nucleus and one or more electrons running around; all matter world consisting of such atoms. Some greater problems were linked with the light that exhibited a kind of duality. There were two different characteristics: slit interference phenomena that were explained by wave behavior, and photoeffect when the light was similar to particle objects. Asymmetry existed

now between these two worlds (matter and light) that were earlier quite different. And the question arose: Is it possible to describe the light interference on the basis of particle characteristics? or: Is it possible to describe both the worlds on similar grounds?

Two different answers were given to these questions. First, W. Duane [24] tried to show that interference phenomena may be explained on particle basis as scattering of individual photons. He assumed that the photons going through a grid obtained perpendicular momentum that was equal to a multiple of h/L where L was the grid constant (i.e., the distance between two slits). It means that the directions of individual photons would be given by perpendicular momenta $p_{\perp} = \frac{h}{L}, \frac{2h}{L}, \dots$, corresponding to measured maximum values in interference picture.

The other attempt was undertaken by L. de Broglie [25] who started in principle from particle aspect of individual objects (similarly as Duane). He combined the idea of Planck with relativistic idea. If a particle has rest mass m_0 its energy is m_0c^2 ; it is then possible to ascribe to it according to Planck the following frequency

$$\nu_0 = \frac{m_0c^2}{h}.$$

As to the frequency corresponding to moving particle L. de Broglie proposed formula

$$\lambda = \frac{h}{p}$$

where p was particle momentum.

The duality idea of L. de Broglie was used as basic idea of further progress. He was convinced that particles represented full reality in the world. However, even if practically all energy was concentrated in particle the wave was linked according to him with each particle; his pilot wave represented a kind of reality, too. The duality idea of L. de Broglie was then strongly supported by interference characteristics found experimentally for electrons passing through a crystal [26].

3.5 Schrödinger equation

The idea of L. de Broglie was made use of by E. Schrödinger who was convinced that all the matter world consisted primarily of waves. In the past many nature phenomena were described with the help of various wave equations; e.g., sound or electromagnetic waves. For a monochromatic wave with the frequency $\nu = \frac{E}{h}$ Schrödinger proposed the following equation

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m_0} \Delta \psi(\mathbf{r}, t) + U(\mathbf{r})\psi(\mathbf{r}, t)$$

He was fully successful as he was able to reproduce practically all results obtained earlier with the help of Hamilton equations. Individual physical quantities were now

represented by operators \mathbf{A} acting on corresponding ψ -functions; it was possible to write

$$A_f(t) = \int dV \psi^*(\mathbf{r}, t) \mathbf{A} \psi(\mathbf{r}, t)$$

where it was integrated over the whole volume and $A_f(t)$ represented time dependence of individual physical quantities.

The solutions of Schrödinger equation could be written in the form

$$\psi(\mathbf{r}, t) = \int dE c(E) e^{-iEt/\hbar} \psi_E(\mathbf{r})$$

where complex function $c(E)$ (determined by initial conditions at time $t = 0$) represented physical characteristics of the given system and functions $\psi_E(\mathbf{r})$ were square-integrable solutions (Hamiltonian eigenfunctions) of time-independent Schrödinger equation

$$H\psi_E(\mathbf{r}) = E\psi_E(\mathbf{r}).$$

H was the Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} + U(\mathbf{r})$$

used standardly in the Hamilton approach, where now $\mathbf{p} \equiv \frac{\hbar}{i} \frac{\partial}{\partial \mathbf{x}}$.

It followed from the preceding that solutions of time-dependent Schrödinger equation might be expressed as linear superpositions of eigenfunctions of the Hamiltonian. They could be represented, therefore, by vectors of the Hilbert space spanned on these eigenfunctions (see [6]). And one should conclude that in the case of stable objects the representation should be equivalent to that in the classical phase space. However, the new representation led to a new (and strange) physics, which will be discussed in following sections to greater details.

3.6 Interpretation of wave function

According to the preceding the Schrödinger equation did not mean anything more than a new formal mathematical approach of calculating physical quantities (without any physical interpretation of the wave function). Such a view changed very much when in 1926 M. Born [3] attributed the meaning of probability density to the wave function. However, the decisive change came when N. Bohr [4] linked the wave function with properties of individual (microscopic) particles. The microscopic world lost its ontological nature as matter objects were interpreted on the basis of characteristics following fully from wave nature.

While for L. de Broglie the waves were closely linked to the existence of corresponding matter objects, the situation changed now. The quantum-mechanical microscopic world exhibited paradoxical properties, being once of particle kind and once of wave kind. Also the individual particles exhibited very strange characteristics being distributed in the space with the density determined by the absolute value

of wave function in the given space point. All matter objects as well as photons were handled, however, in the same way.

A series of physicists were not satisfied with such a physical picture of the microscopic world. However, they abandoned gradually their critical points of view when they were unable to propose a more suitable mathematical model. Thus, Bohr's interpretation became practically the only theory of the microworld, even if the quantum-mechanical world was very strange. The philosophical conviction of the whole society contributed very much to given conclusions.

The only man who remained always critical was A. Einstein. He was denoting the quantum mechanics as incomplete. Even if he was not able to convince physical community to accept his view his objections and his controversy with N. Bohr were discussed during the whole last century and indicated all the time that some points were not satisfactorily answered. Some questions were even denoted as forbidden in the quantum mechanics. The main points of the mentioned controversy will be summarized in the next section.

3.7 Einstein - Bohr controversy

Einstein started to criticize Bohr's Copenhagen quantum mechanics from the very beginning. However, his first arguments were not sufficiently reasoned and remained without any success. The important argument was published by A. Einstein together with two co-authors [27] in 1935.

A Gedankenexperiment (called commonly EPR experiment) was proposed that should have demonstrated the deficiency of the quantum-mechanical model. It was argued that according to orthodox quantum mechanics the measurements performed on two microscopic particles emitted from one object in opposite directions should have been mutually dependent even if the distance between two measurement devices were very large. N. Bohr [28] opposed strongly and succeeded in convincing physical community that such a characteristic belonged to properties of the microworld.

The question was whether the ψ function at one time point described fully all properties of a microscopic particle or whether some other ("hidden") parameters should have been added. The physical community was influenced by the argument of von Neumann [6] that the corresponding extension was excluded by the quantum-mechanical mathematical model.¹ Any attention was not devoted to arguments of Grete Herrmann [29], either, who tried to show that the argument of von Neumann was a circular proof.

The situation started to change partially only when D. Bohm [30] showed that a hidden parameter was contained already in the Schrödinger equation. Bohm proposed also a modification of EPR experiments: instead of measuring the positions

¹Such a conclusion followed from the superposition principle and from the representation of time-dependent solutions of Schrödinger equation in the Hilbert space spanned on one set of Hamiltonian eigenfunctions; the time dependence could not be truly expressed under such conditions. The given problem will be analyzed and corresponding solution will be presented in Part 4.

of involved particles he proposed to emit a pair of particles with opposite spins and to measure their spin orientations; the idea used then practically in all experiments having been performed later. On the basis of Bohm's finding L. de Broglie and J. P. Vigi er returned to the older idea of pilot wave and started to develop the theory of empty waves, carrying the information only, but not any energy (see, e.g., [32]). An additional quantum potential depending on space arrangements of individual experiments (and on initial conditions, too) played a role in such an approach. Particles could move along some definite tracks in such a theoretical alternative.

3.8 Bell's inequalities

The decisive progress (as to Einstein-Bohr controversy) was done when J. Bell [31] showed that one assumption involved in von Neumann's approach was in contradiction to real situation in the nature; for details see, e.g., [32]. He modified this assumption in agreement with reality, which enabled the existence of hidden parameters. He applied the new mathematical model to the EPR experiments in Bohm's modification and argued that some inequalities between measured values should be fulfilled in the theory of hidden variables, while they did not hold commonly in the standard quantum mechanics.

This fact initiated many physicists to look for suitable experiments enabling to solve the controversy between Einstein and Bohr on experimental basis. It was recommended to use the emission of two photons with opposite spins and to measure the coincidence output with the help of two polarizers under different angles. A detailed review of the whole problem may be found in paper of Clauser and Shimony [33].

In the given experiment two photons with opposite spin orientations are emitted in opposite directions and pass through two polarizers:

$$a \mid \longleftarrow o \longrightarrow \mid b .$$

The orientation of transversal spins is randomly and uniformly distributed. Coincidence transmission is then measured at different angles α and β of individual polarizers and corresponding probabilities are established. Let us denote the probability that the left photon is transmitted by a_α and similarly for the right photon – by b_β . When both the photons are fully independent at the moment of measurement four different probabilities should fulfill (according to Bell) the following inequality

$$a_1 b_1 + a_2 b_1 + a_1 b_2 - a_2 b_2 < 2$$

where a_1 and a_2 are average transmission probabilities of the left polarizer for two angles and b_1 and b_2 – the same for the right polarizer.

Such a condition should hold for the given combination of any four (2x2) probabilities while in the case of quantum mechanics it should be violated for some angle combinations. And thus, the physics seemed to be in the position when it should be

possible to decide the controversy between Einstein and Bohr on experimental basis. Any violation of the given condition should prefer the standard quantum mechanics, while in the opposite case the hidden-variable theory would be preferred.

3.9 Orthodox and ensemble interpretations of QM model

While any $\psi(\mathbf{r})$ function should represent the properties of individual microscopic particles its form may be experimentally derived with the help of statistical results in experiments performed with many objects prepared in the same way. I. e., the experimental arrangement corresponds to a wave function while experimental results are given by a statistical distribution of measured values. That was interpreted as a consequence of an "absolute" chance in behavior of microscopic objects, the results being limited only by the predictions following from the quantum-mechanical model.

Such a situation seemed for some physicists to be untenable. They wanted, therefore, to relate the given wave function not to an individual particle but directly to the measured statistical distribution. Consequently, two different interpretations of the quantum mathematical model were discussed: Copenhagen (or orthodox) and ensemble (or statistical); see, e.g., [34]. In the latter case one might practically avoid the mentioned paradoxes (following from the absolute chance in interactions of individual microscopic objects). However, at the same time one should conclude that the given mathematical model corresponds to incomplete description of individual objects; and some "hidden" variables should be added to describe the given situation in a complete way.

The extended mathematical model corresponding practically to the mentioned ensemble interpretation will be described in Part 4.

3.10 Results of EPR experiments

The experiments initiated by Bell's inequalities were performed in principle in 1971-82. Additional experiments performed yet later did not bring anything new. The final results (see L. Aspect et al. [35]) might be formulated in the following way:

- inequalities derived by Bell have been surely violated;
- experimental results have been found to be practically in harmony with quantum-mechanical predictions.

In the next following years these results were often being denoted as victory of quantum mechanics. However, gradually such voices have decreased as it has become always more evident that any of the quantum-mechanical problems has not been solved. And it was necessary to go deeper into basic assumptions on which the whole quantum-mechanical model was built up.

It has been possible to show that the refusal any hidden-variable alternative based on the results of EPR experiments have been strongly influenced by two additional unjustified statements:

- in his book F. Belifante [36] has stated that the Malus law measured for light

transfer through two polarizers and being in harmony with quantum mechanics cannot be derived in the framework of a hidden-variable theory; statement that must be denoted as false (see the next section);

– it has been usually stated that the inequalities derived by Bell have been valid for any hidden-variable alternative as any assumptions have not been involved in their derivation, which is not true, either; see Sec. 3.12.

Both these statements will be analyzed in the next two sections. Hidden-variable model (theory) will be then presented in Part 4 that should be surely preferred to the standard quantum mechanics.

3.11 Malus law for two polarizers

As mentioned the quantum-mechanical interpretation of EPR experiments seems to have been strongly supported by the argument of Belifante who stated wrongly that the predictions of a hidden-variable theory had to differ significantly from those of quantum-mechanical model (see the corresponding graphs on p. 284 in Ref. [36]). According to a hidden-variable theory the transmission of a photon through a polarizer pair (or of two equally polarized photons in coincidence arrangement) should equal

$$p_2(\alpha) = \int_{-\pi/2}^{\pi/2} p_1(\lambda) p_1(\lambda - \alpha) d\lambda$$

where $p_1(\lambda)$ is transmission probability through one polarizer; λ - deviation of photon polarization from the axis of the first polarizer; and α - the angle deviation of the second polarizer. The same formula holds in both the arrangements if the photon polarization does not change in passing through a polarizer.

It has been known from the one-side experiments that it holds

$$p_2(\alpha) = M(\alpha) = (1 - \varepsilon) \cos^2(\alpha) + \varepsilon$$

where for real (imperfect) polarizers $\varepsilon > 0$ (generalized Malus law); ε being very small. Belifante has chosen

$$p_1(\lambda) = \cos^2(\lambda);$$

i.e., he has interchanged quite arbitrarily the transmission through one polarizer and through a pair of them. Malus law may be easily reproduced in the hidden-variable alternative if $p_1(\lambda)$ is chosen in a corresponding way as shown in Fig. 1 (full line); see [37] for necessary details.

And one must conclude that any preference does not follow from experimental data for the quantum-mechanical interpretation of coincidence EPR experiments. Nothing prevents us, therefore, from interpreting all available experiments on the basis of a hidden-variable theory. This fact has opened also a new way to answer the question concerning the time operator (or Pauli's) problem; see Part 4.

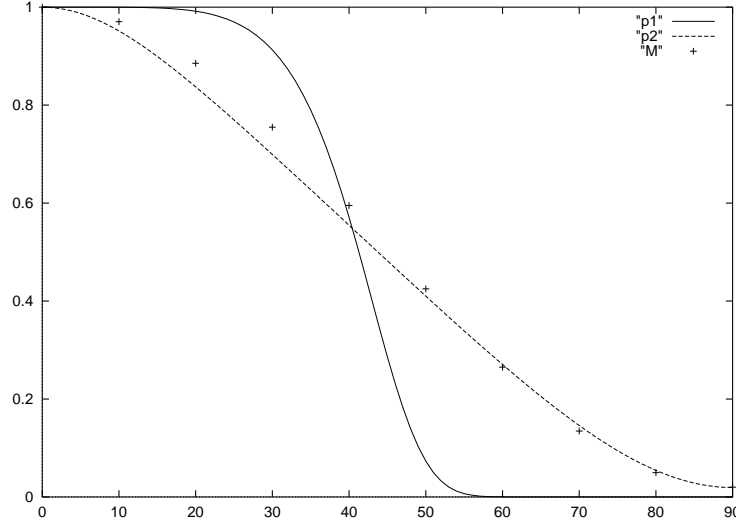


Figure 1: *Transmission probability through a polarizer pair leading to Malus law; $p_1(\lambda)$ - full line; $p_2(\alpha)$ - dashed line; Malus law - individual points.*

3.12 Assumption in derivation of Bell's inequalities

The way to paradoxical interpretation of quantum mechanics has been paved (as already mentioned) by three mistakes: in addition to von Neumann's original mistake two other mistakes have been involved in corresponding argumentations. As the mistake of F. Belifante has been explained in the preceding section we will deal now mainly with the mistake concerning Bell's inequalities.

Even if the mistake of von Neumann has been recognized in principle by Bell its consequences have lasted practically until now, having a close relation to (unphysical) interpretation of general superposition principle. However, the problems have continued from another reason, too, as the approach of von Neumann has been improved by J. Bell only partially. A similar non-realistic (even if weaker) assumption has been involved in derivation of Bell's inequalities, as well. They were being derived in different ways (see, e.g., [33]), but in all of them the same important assumption has been involved, even if sometimes latently.

We will attempt now to explain the essence of this assumption, necessary details being found in [38]. The space characteristics of a photon have not been described completely if it has been characterized by its polarization λ only. It must be regarded (according to photoeffect) as a point-like (very small) particle in any hidden-variable theory and characterized by its position or by its impact parameter \mathbf{f} when coming to a polarizer. And one measures the average coincidence probabilities of photon transfer

$$P_{\alpha,\beta} = \oint d\lambda \oint df_{\alpha} \oint df_{\beta} a_{\alpha} b_{\beta}$$

where \oint represents corresponding averaging integral.

However, to obtain the given inequalities Bell had to make use of the following condition (interchange between divers probability pairs)

$$\oint d\lambda \oint df_\alpha \oint df_\beta a_\alpha b_\beta \cdot a_{\alpha'} b_{\beta'} = \oint d\lambda \oint df_\alpha \oint df_{\beta'} a_\alpha b_\beta \cdot a_{\alpha'} b_{\beta'},$$

which destroys particle property of the photon making the probabilities independent of the value of impact parameter. The same assumption is involved in all kinds (shown in Ref. [33]) of their derivations, even if in some of them it is not explicitly mentioned (see [38]).

It means that the measuring device has been regarded as a black (or at least semi-black) box in a similar way as in the standard quantum mechanics. Actual localization of a photon has been omitted. And one must conclude that the violation of Bell inequalities in coincidence polarization experiments is quite irrelevant as to the support for standard quantum mechanics.²

3.13 Time operator in Hilbert space

In the preceding sections we have discussed the problems related to the controversy between Einstein and Bohr and to the EPR experiments. And we have shown that there is not any argument against Einstein's criticism. Now we will return to the problem of time evolution of physical systems in the framework of the Hilbert space and to the problem of defining time operator.

The evolution of a physical system is described by solutions of time-dependent Schrödinger equation that are then represented in the Hilbert space spanned on one simple set of Hamiltonian eigenfunctions. We have already mentioned that Pauli showed in 1933 that the existence of the time operator fulfilling condition

$$i[H, T] = 1 \tag{1}$$

required for Hamiltonian spectrum to be continuous in the whole interval $(-\infty, +\infty)$.

²In some papers the problem of the maximum expectation value of Bell operator \mathbf{B} (corresponding to the combination of four measured quantities) discussed in Sec. 3.8 has been solved on the basis of the properties of corresponding operators in the Hilbert space. It has been assumed that the Hilbert space has consisted of the product of two subspaces corresponding to individual measuring devices a and b ; measurements of transmission probabilities being represented by operators \mathbf{a} and \mathbf{b} . The given problem has been discussed, e.g., in Ref. [39]. Under different assumptions three numerical limit values have been derived: 2, $2\sqrt{2}$, and $2\sqrt{3}$. The first value (Bell's limit) has been obtained if the commutativity of operators \mathbf{a} and \mathbf{b} has been combined with the additional assumption – the commutativity of operators \mathbf{a} or \mathbf{b} inside individual subspaces – equivalent to the case of Bell and discussed in Sec. 3.12. The other value has been based on the mere mutual commutativity between operators \mathbf{a} and \mathbf{b} , which has corresponded to the basic limit for any hidden-variable alternative (being attributed mistakenly to the standard quantum mechanics in [39]). The actual quantum-mechanical limit should correspond to the case (denoted as unphysical in [39]) when the only limitation has consisted in the requirement for measured probabilities to lie in the interval $(0, 1)$.

We have introduced, too, that there is a kind of contradiction between the time-dependent Schrödinger equation and requirements following from the superposition principle in the Hilbert space. It has been clear already for a longer time that the solution cannot be reached in the framework of the given Hilbert space and that the standard Hilbert space should be extended. However, the way to the solution has been influenced by the problem concerning the exponential phase operator, which will be mentioned now.

The problem of time operator in quantum mechanics started to be solved by P. Dirac [8] for the case of linear harmonic oscillator already in 1927. The oscillator has been described by Hamiltonian

$$H = \frac{p^2}{2m} + kq^2 .$$

It has been possible to introduce operators

$$\mathcal{E} = (aa^\dagger + 1)^{1/2} a , \quad \mathcal{E}^\dagger = a^\dagger (aa^\dagger + 1)^{1/2}$$

where annihilation and creation operators have been defined by

$$a = p - im\omega q, \quad a^\dagger = p + im\omega q, \quad \omega = \sqrt{\frac{k}{m}} .$$

It holds

$$[H, \mathcal{E}] = -\omega \mathcal{E}, \quad [H, \mathcal{E}^\dagger] = +\omega \mathcal{E}^\dagger, \quad \mathcal{E}\mathcal{E}^\dagger = 1 ,$$

and \mathcal{E} corresponds to exponential phase operator defined as

$$\mathcal{E} = e^{-i\omega T} .$$

However, it has been shown later (1966) by Susskind and Glogower [9] that the operator \mathcal{E} is not unitary, but only isometric: $\mathcal{E}^\dagger \mathcal{E} = 1 - |0\rangle\langle 0|$, as it holds

$$\mathcal{E}^\dagger \mathcal{E} u_{1/2} \equiv 0 \neq u_{1/2} .$$

It means that the unitarity condition is not fulfilled for the state vector corresponding to the minimum-energy (vacuum) state.

And the question has been whether this problem may be correlated to the problem of Pauli or not. And how both the problems might be removed: whether by one common extension of the Hilbert space or whether they represent two different problems. The answers to these questions will be given in Part 4. Some additional remarks will be introduced yet in the next section.

3.14 Quantum mechanics and microworld paradoxes

Even if many physicists tried to contribute to the solution of the mentioned quantum-mechanical problems their majority has seemed to be quite content with the given situation. And we should ask what the main reason was.

It was surely the philosophical attitude of the whole human society. The positivistic philosophy convinced the scientists already in the 19th century that it is sufficient for the physical science to represent measured values by mathematical formulas only; to look for any deeper insight into the matter world was even forbidden. Any metaphysical and ontological considerations were being refused. On the other side some (paradoxical) ideas following from contradictory mathematical pictures of the world were being fully accepted (at least by some important physicists). The belief in the quantum-mechanical paradoxes was then strongly supported by far-east philosophies the ideas of which were being widely spread in the western world mainly in the beginning of the 20th century.

Also the bivalent logic of Aristotle was being refused on the basis of earlier conviction. The paradoxical picture of quantum mechanics contributed to calling for many-valent logic. However, there is not more any argument against it when the mentioned mistakes have been removed. It is the full return of Aristotle's bivalent logic into scientific (physical) approaches that might bring new renaissance into the modern fundamental science.

The scientific method is represented mainly by falsification approach. According to it the reliable scientific conclusion may be given by logical contradiction. Only the refusal of a scientific hypothesis (or a set of such hypotheses) may represent actual scientific truth. If no contradiction to experimental data has been found the hypothesis may be denoted as plausible, but it can be never regarded as verified. The scientific method consists in the falsification, there is not any verification in the science.

However, the idea of falsification was devaluated by the ideology of falsifiability refusing any later parallel idea leading to the same results even if it was not falsified and should have been denoted as plausible. It is the falsification principle that requires to look for parallel hypotheses, which opens new ways of knowledge for the future. Forbidding competitive ideas makes looking for truth practically impossible. The idea of falsifiability (as used in physics in the last time) contradicts the falsification method and should be denoted as anti-scientific.

In addition to reasons following from the philosophical attitudes there was also one physically-mathematical conviction (as already mentioned) that contributed significantly to the fact that the problems of quantum mechanics remained unsolved. Practically all physicists trying to solve the two problems (concerning time operator and exponential phase operator) were convinced that both the mentioned deficiencies represented one common problem, that should be solved at the same time, which was not, however, true.

In preceding sections we have analyzed main mistakes on which the arguments

supporting the standard quantum mechanics were based. It has been shown that there is not any argument that would prefer orthodox quantum mechanics to a hidden-variable theory in interpreting experimental data. In the following Part 4 it will be shown how the individual problems may be solved in two different ways; the agreement of the extended mathematical model to experimental data will be also discussed.

4 Extended quantum-theoretical mathematical model

4.1 Schrödinger equation and superposition principle

It is possible to say that the standard (orthodox) quantum mechanics is based on two following basic assumptions:

- any state of a physical system consisting of N objects and its time evolution is represented by the wave function $\psi(\{x_{k,j}\}, t)$ that is obtained by the solution of time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(\{x_{k,j}\}, t) = H \psi(\{x_{k,j}\}, t)$$

where H is corresponding Hamiltonian

$$H = \sum_{j=1}^N \sum_{k=1}^3 \frac{p_{k,j}^2}{2m_j} + V(\{x_{k,j}\})$$

and $\{x_{k,j}\}$ and $\{p_{k,j}\}$ are coordinate and momentum components of individual objects; $V(\{x_{k,j}\})$ is the sum of potential energies between N individual mass objects; it has been put $p_{k,j} = \frac{\hbar}{i} \frac{\partial}{\partial x_{k,j}}$ in the last equation ;

- any physical state is represented by a vector in the Hilbert space being spanned on one set of Hamiltonian eigenfunctions

$$H \psi_E(\{x_{k,j}\}) = E \psi_E(\{x_{k,j}\})$$

and all states (vectors) are bound together with the help of superposition principle.

Let us start with the problem concerning the application of superposition principle in the region of physics. It is possible to show that the general mathematical superposition principle holding for all solutions of linear differential equations has nothing to do with physical reality, as actual physical states and their evolutions are uniquely defined by corresponding initial conditions.³ They characterize individual solutions and represent properties of a physical system, some of them being

³It is being argued that any superposition of two solutions of Schrödinger equation is again a solution of the same equation. However, such a statement is entitled in physics only if both these solutions correspond to the same physical initial conditions (characterized by different ψ -functions). Superposing solutions belonging to divers initial conditions one obtains a solution corresponding again to further fully different initial conditions, which means that significantly divers physical states have been combined in the framework of the standard quantum-mechanical model in a physically unallowed way.

conserved during the whole evolution. And consequently, any physical meaning cannot be attributed to general superpositions in the Hilbert space (going behind superposing the same physical trajectories).

Attributing physical interpretation to superpositions of different solutions (belonging to divers physical states) introduces a drastic assumption into the physics without any proof and any need. Statements that quantum mechanics (including superposition rules) has been experimentally verified must be regarded as wrong, too. All hitherto experimental tests have concerned the time-dependent solutions of Schrödinger equation only (see also [40]).

The other problem consists in the question whether one vector of the given Hilbert space does represent a definite initial condition. If one limits oneself to the physical system consisting of two objects in CMS one can see easily that it is not so in the standard Hilbert space. The wave function $\psi(\mathbf{r}; 0)$ determines initial vectors \mathbf{r}_0 and \mathbf{p}_0 (position and momentum). However, if the Hamiltonian is the function of momentum squared it is evident from the Schrödinger equation that the time derivative of wave function is the same for $\pm\mathbf{p}_0$. Consequently, two different solutions evolving in opposite directions cannot be distinguished in the Hilbert space, in which the sign of the first time derivative is not harmonized with that of \mathbf{p}_0 . The Hilbert space is to be extended and to consist of two different subspaces, the approach being very similar to that proposed by Lax and Phillips [41] in the case of wave equation containing the second time derivative of wave function; the problem being discussed in Sec. 4.2.

In the preceding Part 3 we have described the problems that accompanied quantum mechanics during the whole past century. Now we should like to summarize their possible solutions. Only main ideas and concepts will be explained in this paper; corresponding mathematical structures have been described to greater details already elsewhere (see Ref. [42]).

4.2 Time operator and extended Hilbert space

In the standard quantum-mechanical model the Hilbert space has been spanned on a simple vector basis consisting of Hamiltonian eigenfunctions. In such a case the states corresponding to opposite momenta (of different signs) have exhibited in principle the same evolution (Hamiltonian being defined by momentum squared). Consequently, it has not been possible to represent them by clearly defined trajectories. And Pauli's critique [7] has concerned just this fact.

The given deficiency may be removed if the standard Hilbert space is extended (doubled) in a way, as it was done by Lax and Phillips [41] already in 1967 (see also [43]) and derived independently by Alda et. al. [44] on the basis of the requirement for unstable particles to exhibit purely exponential decay.

Let us demonstrate shortly the given Hilbert structure on the example of a system consisting of two unbound particles. The corresponding Hilbert space consists

of two subspaces:

$$\mathcal{H} \equiv \{\Delta^- + \Delta^+\},$$

each of them being spanned on one set of Hamiltonian eigenfunctions. The individual solutions of Schrödinger equation are then represented by corresponding trajectories in the total Hilbert space.

In the case of continuous Hamiltonian spectrum any point on such a trajectory may be characterized also by expectation values of the operator $R = \frac{1}{2}\{\mathbf{p}, \mathbf{q}\}$, where \mathbf{q} and \mathbf{p} are coordinates and momentum of one particle in CMS. The states belonging to Δ^- are incoming states ($\langle R \rangle < 0$), and those of Δ^+ - outgoing states (independently of the choice of coordinate system). The evolution trajectory representing a solution of Schrödinger equation goes always in one direction from "in" to "out" and is characterized by a set of initial conditions. There is a subset of states ψ_0 characterized by $\langle \psi_0 | R | \psi_0 \rangle = 0$.

It is then possible to introduce also time operator T fulfilling Eq. (1) and initial condition $\langle \psi_0 | T | \psi_0 \rangle = 0$. The subspace Δ^+ (Δ^-) corresponds then also to positive (negative) expectation values of T . The structure of total Hilbert space (corresponding to Schrödinger equation) may be defined by

$$\mathcal{H} = \overline{\bigcup_t U(t)\Delta^-} = \overline{\bigcup_t U(-t)\Delta^+}$$

where

$$U(t) = e^{-iHt} \quad (t \geq 0)$$

is evolution operator. It holds also, e.g.,

$$\Delta^+ \subset \overline{\bigcup_t U(t)\Delta^-}, \quad \Delta^- \subset \overline{\bigcup_t U^\dagger(t)\Delta^+}.$$

As in the case of two colliding particles the two different kinds of states ("in" and "out") may be easily experimentally distinguished it is useful to assume that Δ^+ and Δ^- are two mutually orthogonal subspaces. It is then also possible to join an additional orthogonal subspace Θ that might represent corresponding resonances formed in particle collisions (see [41]-[44]); i.e.

$$\mathcal{H} \equiv \{\Delta^- \oplus \Theta \oplus \Delta^+\}.$$

It is only necessary to define the action of evolution operator between Θ and other subspaces in agreement with evolution operators defined already in individual subspaces Δ^\pm .

The evolution goes in one direction, at least from the global view; some transitions between internal states of Θ may be reversible and chaotic. However, global trajectories tend always in one direction; see the scheme in Fig. 2.

In the case of discrete Hamiltonian spectrum (e.g., harmonic oscillator) the wave function has similar t -dependent form. However, the evolution is periodical as a rule.

The Hilbert space will consist of two subspaces, too (or rather of an infinite series of such pairs if one wants to represent individual subsequent periods); orthogonality between neighbor subspaces being not required. The evolution may be again characterized by trajectories corresponding to different initial conditions.

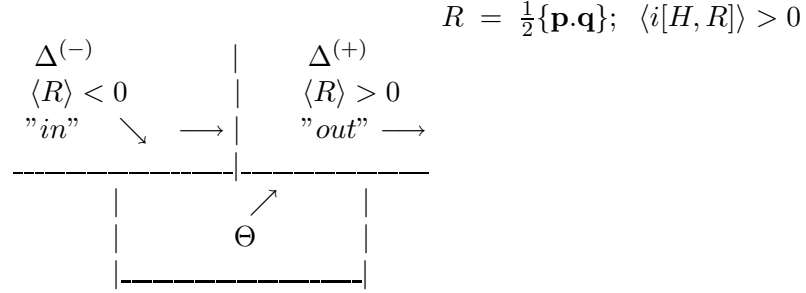


Fig.2: Scheme of the Hilbert space extended according to Lax and Phillips (for a two-particle system); three mutually orthogonal subspaces and direction of time evolution indicated (continuous spectrum).

The expectation values of operator R cannot be used now to define individual points on given trajectories as they change periodically (e.g., faster than $\sin \Phi$). It is, however, possible to introduce time operator T , its zero expectation value being identified with one zero of $\langle R \rangle$; and to characterize individual points on a trajectory by its expectation values. Different points may be characterized, of course, also by expectation values of the phase operator Φ ; or (for one period) in analogy with the continuous case by those of $\tan(\Phi/2)$ or $\cot(\Phi/2)$ increasing from $-\infty$ to $+\infty$. For more details see [42] (or also [40]).

The given extension (doubling) of the Hilbert space has enabled to solve the problem of Pauli. Time operator T is defined regularly in the extended Hilbert space for both the kinds of Hamiltonian spectra and may be expressed as a function of operator R , or of operators \mathbf{p} and \mathbf{q} . Each trajectory represents the evolution of a physical system with given initial conditions (at $t = 0$). Only trajectories corresponding to the same initial conditions may be superposed. The same physical evolution may be pictured by a greater number of trajectories, which introduces a new degree of freedom (or degeneracy), and we might ask whether it does represent also some additional features of reality.

The given question is fully open at the present. It is, however, possible to ask whether this degeneracy might be related to the fact that the colliding particles are complex objects the internal structures of which may be represented by further Hilbert subspaces added in the form of direct products (in the framework of corresponding hidden-variable theory).

The given extension of Hilbert space (according to Lax and Phillips) does not solve, however, the unitarity problem of exponential phase operator. It is possible, e.g., to define creation and annihilation operators for harmonic oscillator in the same

way and with the same consequences as earlier. This problem may be solved in a different way as proposed by Fain [45] already in 1967, too. His proposal remained misunderstood as physicists looked all the time for the common solution with Pauli's problem. The unitarity problem of exponential phase operator will be discussed in the next section (see also [40, 48]).

4.3 Unitary exponential phase operator

The unitarity problem of exponential phase operator has been related practically to one special vector of the Hilbert space, i.e., to the eigenvector corresponding to the minimal eigenvalue of Hamiltonian (zero-energy vector). Fain showed that in the case of linear harmonic oscillator the unitarity of the given operator might be saved if the standard Hilbert space was doubled. Similar solution was then discussed also by R. Newton [46] in 1980. It is, of course, necessary to double the Hilbert space extended already according to Lax and Phillips (or according to Alda et al.); see the preceding section.

This total Hilbert space should consist of two identical mutually orthogonal subspaces:

$$\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_- ,$$

where \mathcal{H}_+ and \mathcal{H}_- are of Lax and Phillips type and are distinguished with the help of special operator J having two quantum numbers $(+1, -1)$. The orthogonality of the subspaces remains conserved during the whole time evolution; it holds (superselection rule)

$$U(t)\mathcal{H}_+ \subset \mathcal{H}_+, \quad U(t)\mathcal{H}_- \subset \mathcal{H}_-.$$

The minimum-energy states in both the subspaces have been mutually linked with the help of the annihilation-type operators. As shown by Fain the states in the two mutually orthogonal subspaces (with separated time evolution) may be distinguished by different signs in the relation between the phase and the flowing time:

$$\Phi = \pm\omega T ;$$

the sign being equivalent to the quantum values of the operator J (cp. [47]).

The question is whether operator J (the sign of the phase) may be interpreted in a physical sense. It follows from the analysis of three-dimensional oscillator [48] that it might be related to the orientation of the co-ordinate system or to the orientation of the corresponding component of resulting spin (or of angular momentum of a two-particle system); see also [42].

4.4 Total Hilbert space and physical reality

Both the problems concerning the regular description of time evolution and the unitarity of exponential phase operator in the framework of Hilbert space have been solved as two different problems by suitable extensions of the standard Hilbert space.

The solution could hardly be reached in the past when the problems were combined together or regarded as one common problem.

The extension proposed by Lax and Phillips is more important and more basic. It might be made use of to represent truly corresponding solutions of time-dependent Schrödinger equation for both the kinds of Hamiltonian spectra (continuous as well as discrete). The operator $U(t)$ represents the evolution going in one direction only; from "in" to "out" in the continuous case, and alternatively from one subspace to another one in the discrete periodical case. A point on individual evolution trajectory may be characterized by expectation values of operator T ; and/or by those of operator R (in continuous case) or of phase operator Φ (resp. $tg(\Phi/2)$) in individual periods (in discrete case – see Sec. 4.2). The given characteristics for the discrete case have been demonstrated on the problem of three-dimensional harmonic oscillator (see [48]) as the simplified picture (in the linear case) does not provide a full answer.

As to the continuous case it is useful to introduce the orthogonality condition between "in" and "out" subspaces as considered by Lax and Phillips. An additional subspace may be included that may represent corresponding unstable resonances formed in collision processes (see Sec. 4.2). The problem will be mentioned yet in Part 5.

To remove the non-unitarity of exponential phase operator the further doubling of the Hilbert structure in addition to that required by Pauli's problem has been necessary. This second doubling differs from the preceding approach in that the evolution in two subspaces remain now permanently separated and there is not any mutual linkage by evolution operator, either; superselection rule holding. The evolution inside both the individual subspaces is then quite identical. However, the question should be put, whether the quantum states belonging to different expectation values of operator J may become evident in some interactions of bound systems of different kinds.

4.5 Hilbert space and classical phase space

We have already mentioned that Schrödinger and Hamilton equations have described a physical system of stable objects in the same way. Also the trajectories in the phase space and in the extended Hilbert space (the first doubling) have exhibited practically the same properties, describing intrinsically irreversible evolution of the given physical system. In this sense the new physical concept of reality is very similar to the case when the extension is based on the rigged Hilbert spaces or complex Hamiltonian eigenvalues (see e.g. [49, 50]). Collision processes and resonance formation (and their decay) may be represented in principle in both the approaches in a similar way.

However, the extension according to Lax and Phillips should be preferred in the case of multi-channel collisions and decays. E.g., inelastic collisions may be easily represented if additional orthogonal "out"-spaces corresponding to different collision

channels are added; all states in individual "out"- and "in"-subspaces being given by solutions of corresponding Schrödinger equations. It is only necessary to define corresponding transition probabilities from an original "in"-subspace to individual "out"-subspaces in agreement with experimental data. The theoretical predictions of such probabilities should be then derived on the basis of concrete collision mechanisms where internal structures of colliding particles should be involved (elastic collisions, fireball formation, stripping, and similarly).

It is also the multi-channel decay of an unstable particle that may be easily included into such a scheme. Such a particle may be represented by a special subspace orthogonal to all "in"- and "out"-subspaces. Such a subspace should possess a greater number of dimensions; the lowest possible dimension number being $N + 1$ where N is the number of decay channels (see [51]). The given particle may be formed as a resonance or as a new particle object produced in inelastic collisions.

The possibility has been now opened, too, to include more systematically the influence of internal structures of individual objects involved in different processes. They may be described by including corresponding subspaces (representing stable or unstable particles) in the form of tensor products; individual subspaces being mutually linked by corresponding evolution operators.

One may say that all research of microscopic objects starts from collisions of two objects. The subspaces representing the structure of individual objects may be more complex (many-dimensional) or very simplified (one vector in extremum) according to the goal of corresponding exploration; i.e., according to the role being played by a given particle in final results. Combining the approaches of Lax and Phillips (and of Fain) may provide a very flexible and adaptable tool for describing and analyzing individual experimental situations.

5 Concluding remarks

According to the orthodox (Copenhagen) quantum-mechanical model the microscopic physical reality should exhibit very strange characteristics, differing rather fundamentally from the classical picture of the world based on earlier (direct) observations and measurements. The new picture has been accepted by physical community even if any sufficient and reasonable explanation has never been given how the observed properties of macroscopic objects may arise from so divers and strange properties of internal (microscopic) components.

Even if the criticism of Einstein was the subject of permanent discussions in the past sufficient attention has not been ever devoted to evident mistakes that have been periodically mentioned. In the preceding we have discussed and analyzed the most important critical points and mistakes and also corresponding solutions of theirs. There is not more any fundamental difference between microscopic and macroscopic world, in contradistinction to what was believed during the 20th century.

Some main points of the new picture should be stressed:

(i) The evolution of the matter world should be regarded as continuous and irreversible change of space arrangements of individual matter structures; including object changes during collision and decay processes. Only special closed systems may evolve periodically (however, not reversibly). Any application of earlier uncertainty principle to individual matter objects represents unacceptable hypothesis.

(ii) The model based on the extended Hilbert space (with narrowed superposition principle) corresponds fully to the quantum-mechanical concept of W. Lamb [52] who seems to regard Schrödinger equation as a natural extension of classical physics to the microscopic world. The model is also in agreement with the recently published results of U. Hoyer [53] who has argued that Schrödinger equation may be derived from classical probability distributions of L. Boltzmann.

(iii) The proposed mathematical model should be preferred not only on the basis of theoretical arguments, but also on experimental grounds. Experiments have been already performed the results of which should be denoted in principle as falsification of the orthodox quantum-mechanical model; see [54, 55, 37].

(iv) The problem of stationary states of bound systems (e.g., atoms) must be newly solved; the Hamiltonian eigenvectors have not more any direct physical meaning, as they do not belong to time-dependent solutions of Schrödinger equation. Other (deeper) reasons, e.g., for the existence of atom orbits and transitions between them, must be looked for.

(v) As to the physical systems consisting of stable objects the use of extended Hilbert space leads to full agreement with classical picture. However, the inelastic collisions and the decay of unstable particle may be now described in the framework of the same mathematical structure. Also the influence of internal structures of involved particles may be easily included into the given scheme; the degree of representation complexity being chosen according to experimental need.

(vi) Superselection rules must be extended into the individual subspaces spanned on double set of Hamiltonian eigenfunctions. Only trajectories belonging to the same initial conditions may be superposed; otherwise, the physical picture might be strongly deformed in comparison with reality.

(vii) The time operator T as well as evolution operator $U(t)$ may be defined in principle regularly in the whole Hilbert space (in all subspaces and for all physically reasoned transitions). An expectation value of operator T may be attributed to any instant state of a given physical system. Such a state is then fully represented by one vector of the whole Hilbert space without exhibiting any strange (paradoxical) properties; ontological nature of the matter world being conserved.

And in the very conclusion: The main milestones of the modern physics have been shown and discussed in the presented paper. We have not gone into mathematical details, referring to corresponding papers where necessary details may be found. The same has concerned the newly proposed mathematical structure. The mathematical details have been omitted, attention being concentrated mainly to corresponding physical ideas. They may be found in corresponding quoted papers

(see mainly Refs. [42, 48]).

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References

- [1] E. Schrödinger: Quantisierung als Eigenwertproblem; Ann. Phys. 79 (1926), 361-76; 489-527; 80 (1926), 437-90; 81 (1926), 109-39.
- [2] E. Schrödinger: Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen; Ann. Phys. 79 (1926), 734-56.
- [3] M. Born: Zur Quantenmechanik der Stossvorgänge; Zeitschr. f. Physik 37 (1926), 863-7.
- [4] N. Bohr: The quantum postulate and the development of atomic theory; Nature 121 (1928), 580-90.
- [5] W. Heisenberg: Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik; Zeitschr. Phys. 43 (1927), 172-98.
- [6] J. von Neumann: Mathematische Grundlagen der Quantenmechanik; Springer 1932.
- [7] W. Pauli: Die allgemeinen Prinzipien der Wellenmechanik; Handbuch der Physik XXIV, Springer Berlin 1933, p. 140.
- [8] P. A. M. Dirac: The quantum theory of the emission and absorption of radiation; Proc. Roy. Soc. (London) A 114 (1927), 243-65.
- [9] L. Susskind, J. Glogower: Quantum mechanical phase and time operator; Physics 1 (1964), 49-61.
- [10] R. Lynch: The quantum phase problem, a critical review; Phys. Rep. 256 (1995), 367-437.
- [11] J. C. Maxwell: On electricity and magnetism; Clarendon Press, Oxford 1873.
- [12] L. Boltzmann: Über die mechanische Bedeutung des zweiten Hauptsatzes der Waermetheorie; Wiener Berichte 53 (1866), 195.
- [13] M. Planck: Über irreversible Strahlungsvorgänge; Ann. Phys. 1 (1900), 69-122.
- [14] A. Einstein: Zur Elektrodynamik bewegter Körper; Ann. Phys. 17 (1905), 891-921.
- [15] P. Lenard: Über die lichtelektrische Wirkung; Annalen der Physik 8 (1902), 149-98.
- [16] E. Ladenburg: Untersuchungen über die entladende Wirkung des ultravioletten Lichtes auf negativ geladene Metallplatten im Vakuum; Ann. Phys. 12 (1903), 558-78.
- [17] A. Einstein: Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt; Ann. Phys. 17 (1905), 132-48.
- [18] E. Rutherford: The scattering of α and β by matter and the structure of the atom; Philosophical Magazine 21 (1911), 669-88.
- [19] N. Bohr: On the constitution of atoms and molecules; Phil. Mag. 26 (1913), 1-25, 476, 857.

- [20] A. Sommerfeld: Zur Theorie des Zeeman-Effekts der Wasserstofflinien, mit einem Anhang über den Stark-Effekt; *Physikalische Zeitschrift* 17 (1916), 491-507.
- [21] N. Bohr, H. A. Kramers, J. C. Slater: Über die Quantentheorie der Strahlung; *Phil Mag.* 47 (1924), 785-802.
- [22] W. Bothe, H. Geiger: Experimentelles zur Theorie von Bohr, Kramers und Slater; *Die Naturwissenschaften* 13 (1925), 440-1.
- [23] A. H. Compton, A. W. Simon: Directed quanta of scattered X-rays; *Phys. Rev.* 26 (1925), 289-99.
- [24] W. Duane: The transfer in quanta of radiation momentum to matter; *Proc. Natl. Acad. Sci.* 9 (1923), 158-64.
- [25] L. de Broglie: Quanta de lumiere, diffraction et interference; *Comptes Rendus* 117 (1923), 548-50.
- [26] C. J. Davisson, L. H. Germer: Diffraction of electrons by a crystal of nickel; *Phys. Rev.* 30 (1927), 705-40.
- [27] A. Einstein, B. Podolsky, N. Rosen: Can quantum-mechanical description of physical reality be considered complete?; *Phys. Rev.* 47 (1935), 777-80.
- [28] N. Bohr: Can quantum-mechanical description of physical reality be considered complete?; *Phys. Rev.* 48 (1935), 696-702.
- [29] Grete Herrmann: Die Naturphilosophischen Grundlagen der Quantenmechanik; *Abhandlungen der Fries'schen Schule* 6 (1935), 75-152.
- [30] D. Bohm: A suggested interpretation of the quantum theory in terms of "hidden variables"; *Phys. Rev.* 85 (1952), 180-93.
- [31] J. S. Bell: On the Einstein Podolsky Rosen paradox; *Physics* 1 (1964), 195-200.
- [32] F. Selleri: Quantum paradoxes and physical reality; *Kluwer Acad. Publ., Dordrecht* 1990.
- [33] J. F. Clauser, A. Shimony: Bell's theorem: experimental tests and implications; *Rep. Prog. Phys.* 41 (1978), 1881-91.
- [34] D. Home, M.A.B. Whitaker: Ensemble interpretations of quantum mechanics. A modern perspective; *Phys. Rep.* 210 (1992), 223-317.
- [35] A. Aspect, P. Grangier, G. Roger: Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A new violation of Bell's inequalities; *Phys. Rev. Lett.* 49 (1982), 91-4.
- [36] F. J. Belinfante: A survey of hidden-variable theories; *Pergamon, Oxford* 1973, p. 284.
- [37] M. Lokajíček: Quantum mechanics and EPR paradox; FZU-D 20020207, Inst. of Phys. AS CR, Prague; <http://xxx.lanl.gov/quant-ph/0211012>.
- [38] M. Lokajíček: Locality problem, EPR experiments and Bell's inequalities; <http://xxx.lanl.gov/quant-ph/9808005>.
- [39] M. Revzen, M. Lokajíček, A. Mann: Bell's inequality and operators' noncommutativity; *Quantum Semiclass. Optics* 9 (1996), 501-6.

- [40] M. Lokajíček: Quantum mechanics and irreversible time flow;
<http://xxx.lanl.gov/quant-ph/0211024>.
- [41] P. D. Lax, R. S. Phillips: Scattering theory; Academic Press, N.Y. 1967.
- [42] P. Kundrát, M. Lokajíček: Irreversible time flow and Hilbert space structures; in: New Research in Quantum Physics (eds. V. Krasnoholovets and F. Columbus), pp. 17-41, 2004 Nova Science Publishers, Inc.
- [43] P. D. Lax, R. S. Phillips: Scattering theory for automorphic functions; Princeton 1976.
- [44] V. Alda, V. Kundrát, M. Lokajíček: Exponential decay and irreversibility of decay and collision processes; Aplikace matematiky 19 (1974), 307-15.
- [45] V. M. Fain: Quantum harmonic oscillator in phase representation and the uncertainty relation between the number of quanta and the phase (in Russian); Zh. Ex. Teor. Fiz. 52 (1967), 1544-8 [Sov. Phys. JETP 25 (1967), 1927].
- [46] R. G. Newton: Quantum action-angle variables for harmonic oscillators; Ann. Phys. (N.Y.) 124 (1980), 327-46.
- [47] M. Bauer: A time operator in quantum mechanics; Ann. Phys. (NY) 150 (1983), 1-21.
- [48] P. Kundrát, M. Lokajíček: Three-dimensional harmonic oscillator and time evolution in quantum mechanics; Phys. Rev. A 67 (2003), 012104; <http://xxx.lanl.gov/quant-ph/0211026>.
- [49] A. Bohm, S. Maxson, M. Loewe, M. Gadella: Quantum mechanical irreversibility; Physica A 236 (1997), 485-549.
- [50] A. Bohm: Time-asymmetric quantum physics; Physical Review A 60 (1999), 861-76.
- [51] M. V. Lokajíček: Resonances as unstable particles with internal degrees of freedom; Czech. J. Phys. B 19 (1969), 1549-55.
- [52] W. E. Lamb, Jr.: Super classical quantum mechanics: The best interpretation of non-relativistic quantum mechanics; Am. J. Phys. 69 (2001), 413-22.
- [53] U. Hoyer: Synthetische Quantentheorie, Hildesheim 2002.
- [54] J. Krása, J. Jiříčka, M. Lokajíček: Depolarization of light by an imperfect polarizer; Phys. Rev. E 48 (1993), 3184-6.
- [55] J. Krása, M. Lokajíček, J. Jiříčka: Transmittance of laser beam through a pair of crossed polarizers; Physics Letters A 186 (1994), 279-82.